Loyola University Maryland

MBA Mathematics Refresher

05/19/2019

Welcome to Loyola's Graduate Business Program! This short guide is aimed at reviewing some of the mathematical concepts MBA students will encounter in the Program. If a more extensive review is desired, The Khan Academy (www.khanacademy.org) has short videos on the web on a wide variety of mathematical and statistical topics. Please don't hesitate to contact Prof. Jeremy Schwartz (jsschwartz@loyola.edu) for further questions.

After each topic in this handout, a sample problem is presented for students to work. Detailed solutions are provided to each problem at the back of this handout starting on page 20.

Reference source for topics covered: *Mathematics with Applications: Finite Version 9th Edition*, by Margaret L. Lial and Thomas W. Hungerford, chapters 1-5.2; plus 6.1 ISBN# 0201770032. Inexpensive copies of this book can be found on amazon.com (the exact edition is not important).

Topics Covered:

A. Percentage Notation

Percents represent parts of a whole, just as fractions or decimals do. Percents are hundredths, or parts of a hundred, and are written with the % sign. One percent means one of one hundred parts. **Note: The decimal 0.05 is not .05%!**

Example: We write 0.75 as a percent by moving the decimal point two places to the right and attaching a % sign to it, 75%.

In statistics, 0.05 is commonly used for probabilities, and it is equivalent to 5%.

Percents are useful because they allow you to compare two pieces of data that may be measured in different units and would otherwise be difficult to compare.

B. Percentage Changes

Question: *Is a percentage increase in price from $3 to $7 the same as the percentage decrease in price from $7 to $3? It is not, please see below for more information.*

Use the formula: Percentage change = [(New price – Old price)/Old Price] *100

Note: The denominator is always the old (or original) price and it is the reference point you use to determine how much the price has changed. Be sure to follow the parentheses in the formula by subtracting first, then dividing, and finally multiplying by 100 to put your answer into percentage terms.

Practice Questions:

1. If the price of your product rises from $3 to $7, what is the percentage increase in price?

2. If the price of your product falls from $7 to $3, what is the percentage decrease in price?

C. **The Distributive Property**: **a*(b+c) = ab + ac**

Examples:

a. 4(-2 + 5) = -8 + 20 = 12
b. 5x + 3x = 8x
c. (8 – k)m = 8m – km

D. **Order of Operations** – when working problems, what do you do first?

1. Work separately above and below the fraction bar.
2. Start with the innermost set of parentheses or brackets, working your way outward.
3. If no parentheses are present:
   a. Find all powers and roots, working from left to right.
   b. Perform multiplication and division next, from left to right.
   c. Perform additions and subtractions last, from left to right.

Example: $\dfrac{-9(-3) + 5}{3(-4) - 5(4)} = \dfrac{27 + 5}{-12 - 20} = 32/-22 = -16/11$

E. **Exponents** –

Natural Number: the positive integers (whole numbers) 1,2,3, etc. and sometimes zero (no negatives and no fractions)
Real Number: Positive or negative whole numbers or decimal numbers

If n is a natural number and a is any real number, then

a^n denotes the product: a * a * a * a * (n factors)
and n is the exponent, a is the base. We say "**a to the nth power**".

Example: $5^2 = 5 * 5$ or $(-5)^3 = (-5)*(-5)*(-5) = -125$

If m and n are numbers, and a is a real number, then **$a^m * a^n = a^{m+n}$**

Example: $(5p)^2 * (5p)^8 = (5p)^{10}$

A **polynomial** in one variable x is an expression of the form
$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x^1 + a_0$

Example: Variable Costs = $-3x^2 + 3480x - 325$ is a parabola (2nd power)

Equations

An **equation** is a statement that says two algebraic expressions are equal. For example in the *equation* 5+2 = 8-1, the *equation* says that what is on the left (5+2) is equal to what is on the right (8-1). To solve an equation is to find all values for its variables (a symbol used for a number we don't know yet – often times it is a letter like x or y) for which the equation is true. When practical applications are stated in mathematical symbols this is called modeling. One of the frustrations of word problems is that the equation (or the model) is not specified in a formula but must be deduced from the verbiage.

Depreciation Equations:
If you purchase an item for business use, in preparing your income tax you may be able to spread out its expense over the life of the item. This is called **depreciation**. One method of depreciation is **straight-line depreciation**, in which the annual depreciation is computed by dividing the cost of the item, less its estimated salvage value, by its useful life. Suppose the cost is C dollars, the useful life is N years, and there is no salvage value. Then the value V (in dollars) of the item at the end of n years is given by the formula:

$$V = C(1 - n/N)$$

Practice Question:

3. Suppose new office furniture is purchased for $1600, has a useful life of 8 years, and has no salvage value. After how many years will it have a value of $1000?

   Helpful hint: It is best to first identify all of the numbers given in the question and determine what you are trying to solve for in the problem. Here, C = $1600, V = $1000, and N = 8, so we are solving for "n".

Accounting and Economics Equations:

Total Cost = Total Variable Cost + Total Fixed Cost
Variable costs are those costs that vary depending on a company's production volume; they rise as production increases and fall as production decreases. **Variable costs** differ from **fixed costs** such as rent, advertising, insurance and office supplies, which tend to remain the same regardless of production output. **Marginal cost** is the cost of one additional unit, and it does not include fixed cost.

Average Cost = Total Cost / number of units **(Note: this cost takes fixed cost into account)**

Total Revenue = (Price/unit)*(number of units)
Total revenue is the total receipts from sales of a given quantity of good or services.

Profit = Total Revenue - Total Cost
Profit is the surplus remaining after total costs are deducted from total revenue.

Practice Question:

4. The Geometric Products Company produces a product at a variable cost per unit of \$2.20. If fixed costs are \$95,000 and each unit sells for \$3, how many units must be sold for the company to have a profit of \$50,000? Hint: Let Q be the number of units sold.

   Variable Cost per unit (Marginal Cost) = \$2.20
   Price = \$3.00
   FC = \$95,000
   Profit = \$50,000

   To start solving the problem, begin with the formulas above. Remember to use the distributive property with the profit function:

   Total Cost = Total Variable Cost + Total Fixed Cost = 2.20*Q + 95,000
   Total Revenue = 3* Q
   Profit = Total Revenue – Total Cost = 3*Q – (2.20*Q + 95,000)

Compounding Interest Equation:

Compound interest is interest added to the principal of a deposit or loan so that the added interest also earns interest from then on. This addition of interest to the principal is called compounding. This is something you will be learning much more about in your finance classes, but a base is needed for the program.

The formula for compounding interest has four key components: FV (the future value), i (the interest rate per period), t (the number of compounding periods), and P (the present value of the future amount or the principal, initial investment).

$$FV = P(1 + i)^t$$

Practice Question:

5. In 2 years a company will begin an expansion program. It has decided to invest \$2,000,000 now so that in 2 years the total value of the investment will be \$2,163,200, the amount required for the expansion. What is the annual rate of interest, compounded annually, that the company must receive to achieve its purpose?

   P = Initial investment (principal) = \$2,000,000
   r = annual rate of interest (variable to be solved for here)
   t = time from initial investment in years = 2
   FV = future value, here the value of the investment after 2 years = \$2,163,000

Equation for Computing Present Value:

Present value (PV) is the current worth of a future sum of money or stream of cash flows given a specified rate of return. Taking the equation for Future Value (FV) above and rearranging the terms to solve for P or PV, we find the formula:

$$PV = FV/(1 + i)^t$$

This formula says that PV is the amount of money I need to deposit today to have FV dollars after t periods at interest rate i.

An example of this formula being used is: A zero coupon bond with face value of $15,000 and a 6% interest rate compounded annually will mature in 5 years. What is a fair price to pay for the bond today? The answer would be $PV = 15{,}000 / (1 + .06)^5 = \$11{,}208.87$.

A second example of this formula being used is in the following practice question:

Practice Question:

6. Suppose you will be receiving payments of $1,000 annually for the next 3 years (3 payments). If the rate of interest is 4%, what is the sum of these three payments worth today? (Note: Why is the answer not $3,000? This is because the interest is not factored into the 1,000 payments).

Inequalities

Two numbers are either identical or one is larger than the other. An **inequality** is a statement that one number is less than another number. (Number line).

- There are symbols that show in what way things relate to one another:
  $a<b$ says that a is **less than** b
  $a>b$ says that a is **greater than** b
  $a\leq b$ means that a is **less than or equal** to b
  $a\geq b$ means that a is **greater than or equal** to b
  $a=b$ means that a is **equal to b**

- If the same constant is added to both sides of an inequality then the order (the sense) of the inequality is the same. $a>b$ then $a+c>b+c$
- If both sides of an inequality are multiplied (or divided) by a **positive** constant, then the order **remains the same**. If $c>0$ and $a>b$ then $ac>bc$.
- If both sides of an inequality are multiplied by a **negative** number, then the order **is reversed**. If $c<0$ and $a>b$ then $ac<bc$.
- If both sides of an inequality are positive and we raise each side to the same positive power then the order is maintained. If $a>b>0$ then $a^n>b^n$ for all n.
- A **closed** interval is an interval where $a\leq x\leq b$, i.e., the endpoints of the interval **ARE** included. An **open** interval DOES NOT include the endpoints, $a<x<b$.

Practice Questions:

7. Each month last year, a company had earnings that were greater than $37,000 but less than $53,000. If S represents the total earnings for the year, describe S by using inequalities.

8. The Davis Company manufactures a product that has a unit selling price of $20 and a unit cost of $15. If fixed costs are $600,000, determine the least number of units that must be sold for the company to have a profit.

<u>Summation Notation</u>:

The Greek capital letter *sigma*, Σ, is the symbol used to show summations of the expression which follows it. It is very important to perform the operations within the parentheses before moving to the operation outside the parentheses!

$\sum_{i=l}^{n} x_i = \mathbf{x_1 + x_2 + ... + x_n}$

$\sum x_i y_i = \mathbf{x_1y_1 + x_2y_2 + ... + x_ny_n}$

Practice Question:

9. Suppose x and y have the following 6 <u>paired</u> <u>values</u> of x and y (i running from 1 to 6):

i	:	1	2	3	4	5	6
x	:	3	5	7	2	4	6
y	:	2	4	6	8	10	12

a. $\Sigma(x_i^2) =$

b. $(\Sigma x_i)^2 =$

c. $\Sigma x_i y_i =$

Functions and Graphs

A. Functions

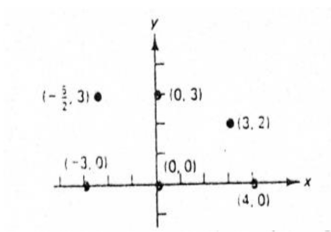
- A **function** is a rule that assigns to each input number exactly one output number. The set of all input numbers to which the rule applies is called the **domain**. The set of all output numbers is called the **range**.
- The input variables are called the **independent** variables. A variable representing the output variable is called the **dependent** variable.
- For each value of the independent variable, the dependent variable can have only one value.
- The notation f(x), read, "f of x", denotes a function of the input variable x. Example: $y=f(x)=x+2$.
- Functions are important in business because they can be used to calculate production costs, determine pricing, measure profits, analyze finances, etc.

Practice Question:

10. A business with an original capital of $10,000 has income and expenses each week of $2000 and $1600, respectively. If all profits are retained in the business, express the value V of the business at the end of t weeks as a function of t. (Hint: Time is the **independent** variable and the value **depends on the time period**.)

B. Graphs in Rectangular (Cartesian) Coordinates

- A **plane** defined by two real number lines that are constructed perpendicular to each other such that their origins coincide is called a coordinate system. A plane is a flat surface and goes on forever. Below is an example of a plane.



- The horizontal axis is labeled with the independent variable.
- The vertical axis is labeled with the dependent variable.
- Any point can be identified by referring to its x value (x coordinate) **and** to its y value (y coordinate), i.e. the (x, y) ordered pair.

C. <u>Logarithms (Base 10 is “log” and base e is “ln”)</u> - *Commonly used business function*

Logarithms answer the question “How many of one number do we multiply to get another number?” For example, how many 3s need to be multiplied to get 27. The answer is 3x3x3=27, so we had to multiply 3 of the 3s to get 27. Thus, we say the logarithm of 27 with base 3 is 3. So $\log_3(27) = 3$.

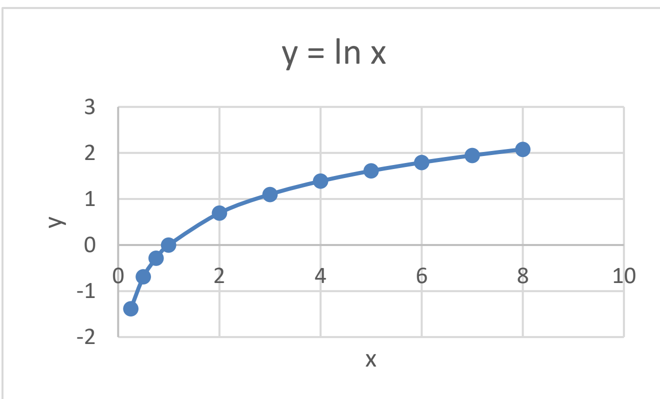
Logarithms are used in mathematics to change the scaling of an axis to be able to see it in its entirety and to help solve financial problems with exponents.

The number **ln x** is the exponent to which e (remember e = 2.71828…) must be raised to produce the number x. So relating this to the above, $\log_e x = \ln(x)$. If we want the number e needs to be raised to, to get 0, that answer would be 1. Because e to the zero power is 1. So ln 1 = 0.

If you want to solve ln x =5, it would be $e^5 = x$, which is (2.71828 to the fifth) 1.48.413.

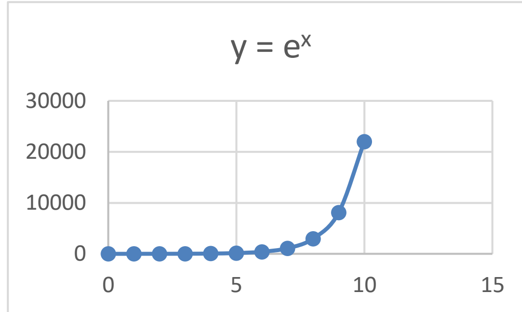
$y = \ln x$ means $e^y = x$.

Here’s the graph for y = ln(x), and you can see that this concept might be used for a situation where more and more of x brings about smaller and smaller increases in y. The phrase “diminishing returns” might fit here.

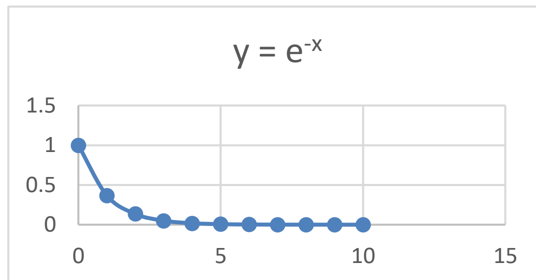


D. Exponential Functions: $y = e^x$

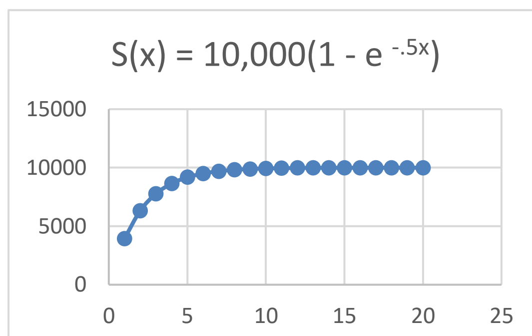
This function is used to depict ever increasing changes in y as x is increased. You may have heard the term, "**exponential growth**", for example. (Compound interest is another example.)



If, as x increases, y is diminishing rapidly to practically zero, we might use the exponential function with a **negative** exponent. For example, infant mortality rates fell dramatically in the U.S. between 1920 and 2000, so an exponential function with a **negative** exponent might fit this situation best.



Finally, when a quantity changes exponentially, but does **not** either grow very large or decrease practically to zero, we need other variations on this function's form.



Example: Suppose sales of a new product often grow rapidly at first and then begin to level off with time. Suppose the annual sales of an inexpensive new gadget is given by the following model:

$S(x) = 10{,}000(1 - e^{-.5x})$

Where x = 0 corresponds to the year when the gadget went on the market. Note the negative exponent. We can explore when sales might level off here with a spreadsheet.

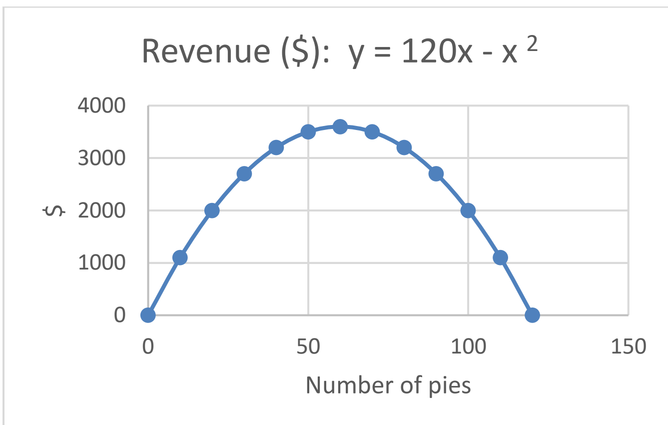
E. <u>Quadratic functions (often called parabolas) with the form $y = a + bx + cx^2$</u>

A quadratic equation is one of the form y: $a+bx+cx^2$ where a, b, and c are number not equal to zero. The graph of a quadratic function is a curve called a parabola. Parabolas may open upward, downward and vary in width or steepness, but they all have the same basic "U" shape.

Quadratic functions are members of the polynomial family of functions with the highest exponent of 2 in the function. This form is often used in economics for **total revenue** and other concepts.

Example of Maximum Revenue*:*

*Maximum revenue for Maxie's Pie Company occurs at the "vertex" of the function below. The vertex of the parabola (which is always symmetrical) is (h, k), where h = -b/2a and k = f(h). Here the vertex would be at the peak, and using the function below, a = -1, b = 120 and c = 0. The "h" is = -b/2a = -120/-2 = 60 pies, and the k is the max revenue is f(60) = 120*60 – 60^2 = $,3600.*



F. Straight Line Functions

Straight lines can be presented in equations in various forms, but the most common form is:
$y = mx + b$, where b is the y-intercept and m is the slope.

The y-intercept is where the equation intersects the y-axis. The amount of vertical change (the change in y) divided by the amount of horizontal change (the change in x) is a measure of the steepness of the line and is called the slope.

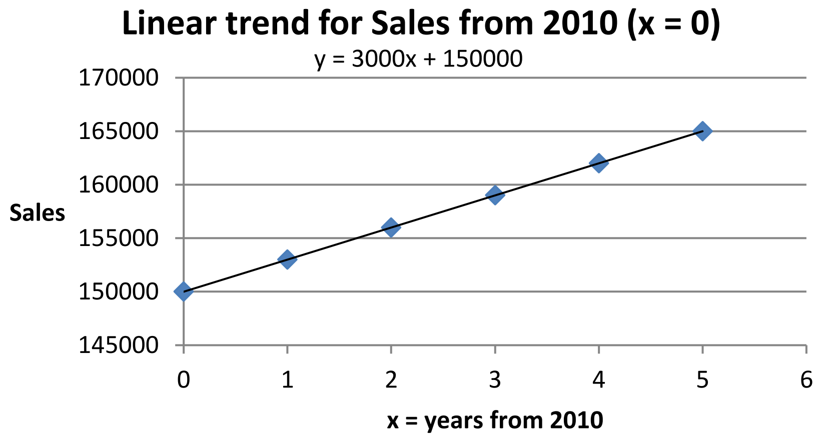
slope = $m = (y_1 - y_2) / (x_1 - x_2)$ = vertical change/horizontal change

- Horizontal lines have zero slope and are written $y = b$.
- Vertical lines have undefined slope and are written $y = a$.
- A line that rises from left to right has a positive slope.
- A line that falls from left to right has a negative slope.
- Two lines are parallel if they have the same slope or are vertical.

Practice Questions:

11. Sketch the graph for the line $y = 3x + 5$ and determine the y-intercept and slope.

12. The projected annual sales S (in dollars) of a new product is given by the equation: $S = 150{,}000 + 3000t$ (where t is the time in years from 2010). Such an equation is called a **trend equation.**



a. Find and interpret **the y-intercept** for management.

b. Find and interpret **the slope** of the line for management.

c. Find the projected sales for 2015.

G. <u>Systems of Linear Equations</u>

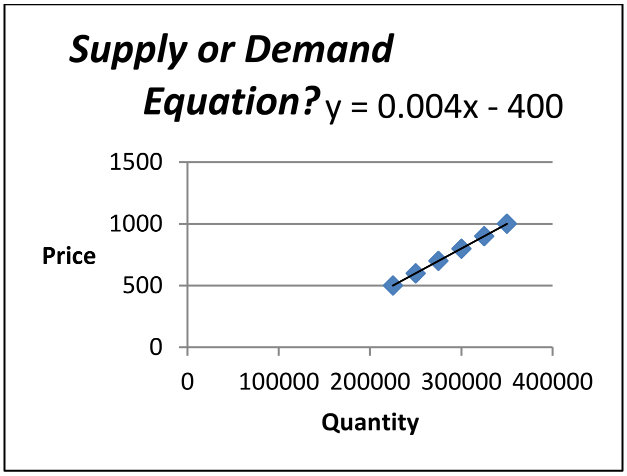
When a situation must be described mathematically, it is not unusual for a set of equations to arise. For example in describing the market for a product, there is an equation describing the quantity demanded at each price for the product from consumers, and an equation describing the quantity supplied at each price of the product from producers. A set of equations is called a **system of equations**. The problem is to find values of x and y for which the equations are true. These values are the solutions of the equations. Potentially, there may be no solution as the lines are parallel. There may be exactly one solution. Or there may be an infinite number of solutions if the two lines are identical. Algebraically, a system of equations can be solved by successively creating an equivalent system such that one of the variables is eliminated.

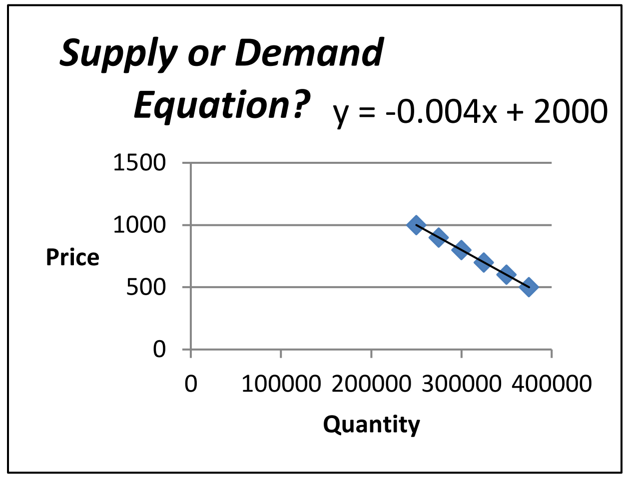
Example 1:

Supply and demand equations for a certain product are (where P represents the price per unit in dollars and Q represents the number of units per time period):

$P = 0.004Q - 400$
$P = -0.004Q + 2000$

Question: Which of the graphs below represents the supply equation and which one represents the demand equation?

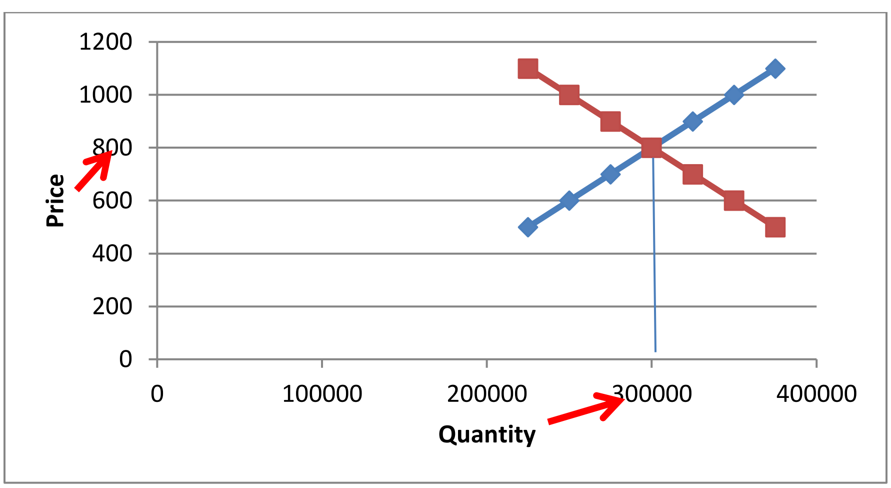




The **supply** equation is the one with the **positive slope of 0.004**. When price rises, firms are willing to supply more units to the market, holding all else constant. Hence, the rise and the run are **positive** when both price and quantity rise (or both negative when both price and quantity fall). This is called a **direct** relationship.

The **demand** equation is the one with the **negative slope of -0.004.** When price rises, consumers cut back on the quantity demanded and search for substitutes. Hence, the price and quantity demanded move in opposite directions; when price falls, quantity demanded is expected to rise, and vice versa. This is called an **inverse** relationship.

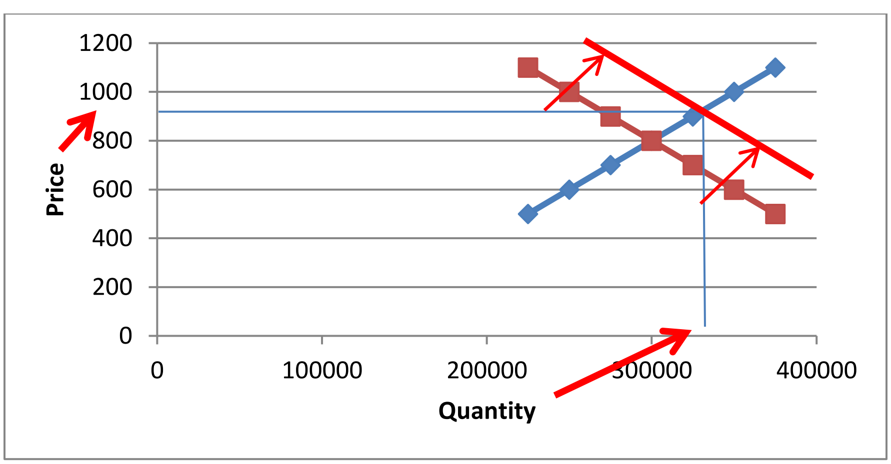
The **equilibrium price** and **quantity** (where supply and demand cross) is visually illustrated below. This price is called **the market clearing price** because at this price there is no excess quantity supplied or demanded.



Practice Question:

13. Find the equilibrium price and quantity for the above. The substitution method is the easiest way to this problem.

If a factor other than the price of this product changes, we can model what we expect will happen on this graph even though the factor is not shown on the x-axis or the y-axis. For example, if incomes changes, then the quantity demanded of this product will change at each and every price. Hence, the demand curve will **shift to the right or to the left** depending on the product in question. If incomes rises, and the product is cruises, then we expect to see more cruises demanded at every price. The demand curve will shift to the right. We expect more cruises to be demanded and a higher price, holding all other factors constant.



Example 2: Two equations and two unknowns – how do we solve them?

<u>QUESTION</u>: Suppose Mardi received an inheritance of $60,000. She would like to invest part at 12% and deposit the remainder in tax free bonds earning 8%. Her total annual income from the investments needs to be $6,800. What is the amount that she should invest at 12%?

In this situation, it's best to lay out everything that is known about the situation at hand and put as much of it into mathematical terms as possible. In this situation, let's identify the following:

> X = the total amount invested at 12%
> Y = the total amount invested in tax free bonds earning 8%

We know that X + Y must equal $60,000, so that is our first equation: X + Y = 60,000

The second equation revolves around the information dealing with the annual income stream from the investments:

> From investment X, she will gain 0.12*X in income.
> From investment Y, she will gain 0.08*Y in income.
> Together these two sources of income should add up to $6,800.
> Written mathematically: 0.12*X + 0.08*Y = 6,800.

We now have two equations and two unknowns and we can solve using the substitution method.

> Since X + Y = 60,000, we know that Y = 60,000 – X

We substitute (60,000 – X) in the second equation for Y and solve for X:

> 0.12*X + 0.08*(60,000 – X) = 6,800

Simplifying we get: .12X + 4,800 - .08X = 6,800

Reducing further: .04X = 2,000

Dividing both sides by 0.04, we have X = $50,000.

<u>ANSWER:</u> Mardi should invest $50,000 in the account earning 12%, and $10,000 in the account earning 8%.

Practice Question:

14. Harvey had $55,000 to invest. He chose to invest a portion in an annuity that had a 6% annual return. The rest of his principal was invested in oil skimmers with a projected 3% annual return. Harvey expects an annual income of $2,600 from these investments. How much should he put into each investment?

1. If the price of your product rises from \$3 to \$7, what is the percentage increase in price?

   Percentage change = [(New price – Old price)/Old Price] *100
   = [(7 – 3) / 3]*100 = 133.33%

2. If the price of your product falls from \$7 to \$3, what is the percentage decrease in price?

   Percentage change = [(New price – Old price)/Old Price] *100
   = [(3 – 7) / 7]*100 = -57.14%

3. Suppose new office furniture is purchased for \$1600, has a useful life of 8 years, and has no salvage value. After how many years will it have a value of \$1000?

   Note: It is best to first identify all of the numbers given in the question and determine what you are trying to solve for in the problem. Here, C = \$1600, V = \$1000, and N = 8, so we are solving for "n" years.

V = C (1- n/N) *C is 1600, V is 1000 and N is 8 so….*
1000 = 1600 (1 – n/8) *work toward isolating the variable by dividing both sides by 1600*
1000/1600 = (1 – n/8) *eliminate parenthesis and simplify fraction on left side of equation to 5/8*
5/8 = 1 – n/8 *isolate variable on one side & make variable positive by + n/8 and – 5/8 to both sides*
n/8 = 1 – 5/8 *eliminate variable fraction by multiplying 8 to both sides*
n = 8*(1 – 5/8) *simplify within parenthesis, converting 1 to a fraction*
n = 8*(8/8 – 5/8) *simplify and solve*
n = 8*(3/8) = 24/8
n = 3 years

4. The Geometric Products Company produces a product at a variable cost per unit of \$2.20. If fixed costs are \$95,000 and each unit sells for \$3, how many units must be sold for the company to have a profit of \$50,000? Hint: Let Q be the number of units sold.
   Cost per unit = \$2.20 Price = \$3.00 FC = \$95,000 Profit = \$50,000

   Profit = Total Revenue – Total Cost
   50,000 = 3 * Q – (95,000 + 2.20*Q)
   50,000 = 3Q – 95,000 – 2.2Q
   50,000 + 95,000 = 0.8Q
   Q = 145,000/0.8 = 181,250

5. In 2 years a company will begin an expansion program. It has decided to invest \$2,000,000 now so that in 2 years the total value of the investment will be \$2,163,200, the amount required for the expansion. What is the annual rate of interest, compounded annually, that the company must receive to achieve its purpose?

P = Initial investment (principal) = \$2,000,000
r = annual rate of interest (variable to be solved for here)
t = time from initial investment in years = 2
FV = future value, here the value of the investment after 2 years = \$2,163,000

$FV = P^* (1 + r)^t$

Solution:

$2,163,000 = 2,000,000(1 + r)^2$	*reduce constant & coefficient by 1,000 to simplify*
$2,163 = 2,000 (1 + r)^2$	*divide both sides by 2000*
$2,163/2,000 = (1+r)^2$	*simplify the fraction and convert to decimal*
$1.0815 = (1 + r)^2$	*eliminate the exponent by taking square root of both sides*
$\sqrt{1.0815} = 1 + r$	*isolate r as the variable on one side*
$r = \sqrt{1.0815} – 1$	*solve*
$r = 1.04 – 1 = .04$ or 4%	

6. Suppose you will be receiving payments of \$1,000 annually for the next 3 years (3 payments). If the rate of interest is 4%, what is the sum of these three payments worth today ("*the present value*")? (Note: Why is the answer not \$3,000? Because the time value of money needs to be factored into the 1,000 payments).

   $PV = 1,000 / (1 + .04)^1 + 1,000 / (1 + .04)^2 + 1,000 / (1 + .04)^3$ (approx.. \$2,780.30)

7. Each month last year, a company had earnings that were greater than \$37,000 but less than \$53,000. If S represents the total earnings for the year, describe S by using inequalities.

   $12 * 37,000 < S < 12 * 53,000$

8. The Davis Company manufactures a product that has a unit selling price of \$20 and a unit cost of \$15. If fixed costs are \$600,000, determine the least number of units that must be sold for the company to have a profit.

   Profit = Total Revenue – Total Cost > 0

   $20 * Q - 600,000 – 15*Q > 0$
   $5*Q > 600,000$
   $Q > 120,000$

9. Suppose x and y have the following 6 paired values of x and y (i running from 1 to 6):

i :	1	2	3	4	5	6
x :	3	5	7	2	4	6
y :	2	4	6	8	10	12

1. $\Sigma x_i^2 = 3^2 + 5^2 + 7^2 + 2^2 + 4^2 + 6^2 = 139$

2. $(\Sigma x_i)^2 = (3 + 5 + 7 + 2 + 4 + 6)^2 = 729$

3. $\Sigma x_i y_i = (3*2) + (5*4) + (7*6) + (2*8) + (4*10) + (6*12) = 196$

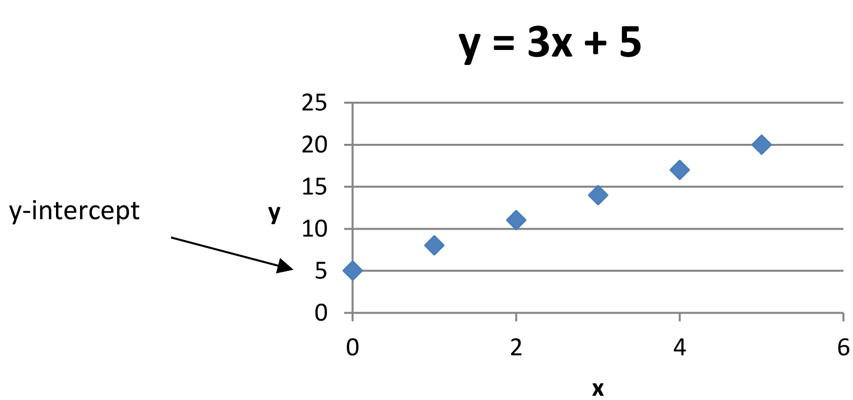
10. A business with an original capital of $10,000 has income and expenses each week of $2000 and $1600, respectively. If all profits are retained in the business, express the value V of the business at the end of t weeks as a function of t. (Hint: Time is the **independent** variable, and the value **depends on the time period**.)

$V(t) = 10{,}000 + (2{,}000 - 1{,}600)*t = 10{,}000 + 400*t$

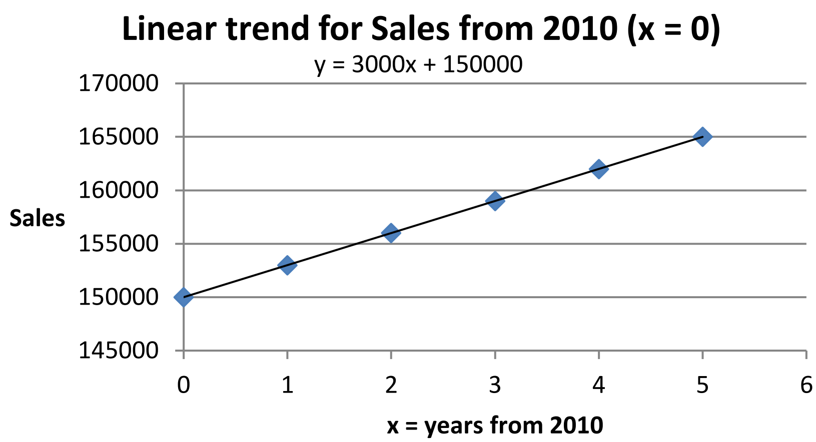
11. Sketch the graph for the line y = 3x + 5 and determine the y-intercept and slope.

(Y-intercept is 5, slope is 3)

Solution: Note that m = 3 in the form y = mx + b, and the slope is 3. For each unit x increases, y increases by 3 units. Likewise, for each unit x decreases, y decreases by 3 units. The slope is positive, indicating that x and y move in the same direction.



12. The projected annual sales S (in dollars) of a new product is given by the equation:
$S = 150{,}000 + 3000t$, where t is the time in years from 2010. Such an equation is called a trend equation.



a. Find and interpret the y-intercept.

Solution: The y-intercept is 150,000 and it represents sales when t = 0. (Plug in zero for t, and you'll see how the 3000t drops out of the equation.)

Interpreting the y-intercept: We first need to determine what year t = 0 represents. Since t is the number of years from 2010, the zero represents the year 2010.

The interpretation for the y-intercept: Sales were estimated to be 3000 in 2010. Now, since 2010 is behind us, the interpretation of the y-intercept is obviously not helpful, but the y-intercept helps calibrate the line so that years into the future can be forecast.

b. Interpret the slope of the line for management.

The slope = 3000. This means that annual sales are forecast to RISE (positive slope!) by 3,000 units each year.

Rise/Run = 3,000 sales/1 year

c. Find the projected sales for 2015.

Solution: 2015 is 5 years from 2010, so t = 5.
Sales in 2015 = 150,000 + 3,000(5) = 165,000

13. Find the equilibrium price and quantity using the "elimination by substitution" method.

Solution: $P = 0.004Q - 400$
$P = -0.004Q + 2000$

If we plug the second equation in for P, we have one equation in terms of Q:

$$-0.004Q + 2000 = 0.004Q - 400$$

Moving all of the "Q" terms to the left side and all of the constants to the right side:

$$2000 + 400 = 0.004Q + 0.004Q$$
$$2400 = 0.008Q$$
$$Q = 2400/0.008$$
$$Q = 300{,}000$$

Find the equilibrium price by plugging Q = 300,000 into **either** the demand or supply equation:

$$\text{Price} = -0.004Q + 2000 = -0.004(300{,}000) + 2000$$
$$= \$800$$

$$\text{Price} = 0.004Q - 400 = 0.004(300{,}000) - 400 = \$800$$

14. Harvey had $55,000 to invest. He chose to invest a portion in an annuity that had a 6% annual return. The rest of his principal was invested in oil skimmers with a projected 3% annual return. Harvey expects an annual income of $2600 from these investments. How much should he put into each investment?

We always start by writing down in mathematical terms as much as we can from the information given to us. Here, we will label:

X = the total amount invested at 6%
Y = the total amount invested in oil skimmers earning 3%

And our first equation is: $X + Y = 55{,}000$

The second equation revolves around the information dealing with the annual income stream from the investments:

Together these two sources of interest income should add up to $2600.
Written mathematically: $0.06*X + 0.03*Y = 2600$.

We now have two equations and two unknowns and we can solve using the substitution method.

Since X + Y = 55,000, we know that Y = 55,000 – X

We substitute (55,000 – X) in the second equation for Y and solve for X:

$$0.06*X + 0.03*(55{,}000 - X) = 2600$$

Rearranging terms, we have: $(0.06 - .03)X = 2600 - (0.03*55{,}000)$

Reducing further, we have: $0.03*X = \$950$

Dividing both sides by 0.03, we have X = $31,667. (rounded)

Harvey should invest $31,667 in the account earning 6%, and $23,333 in the account earning 3%.